

In[152]:= **b[a_] := Sqrt[a² - 1]**
R1[x_, a_, w_] := $\frac{1 - w * (a - b[a] \text{Coth}[x])}{a - w + b[a] \text{Coth}[x]}$

R1[x, a, w]
 Out[154]= $\frac{1 - w (a - \sqrt{-1 + a^2} \text{Coth}[x])}{a - w + \sqrt{-1 + a^2} \text{Coth}[x]}$

In[155]:=

In[156]:= **R1[x, 1, w]**

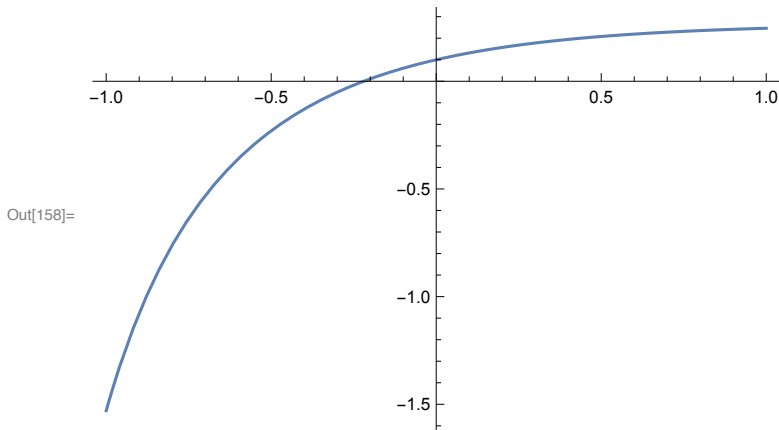
Out[156]= 1

In[157]:=

FullSimplify[R1[x, a, w]]

Out[157]= $\frac{1 - a w + \sqrt{-1 + a^2} w \text{Coth}[x]}{a - w + \sqrt{-1 + a^2} \text{Coth}[x]}$

In[158]:= **Plot[R[x, 2.0, 0.1], {x, -1, 1}]**



In[159]:=

In[160]:=

In[161]:=

c[x_, a_] := a Sinh[x] + b[a] Cosh[x]

R[x_, a_] := Sinh[x] / c[x, a]

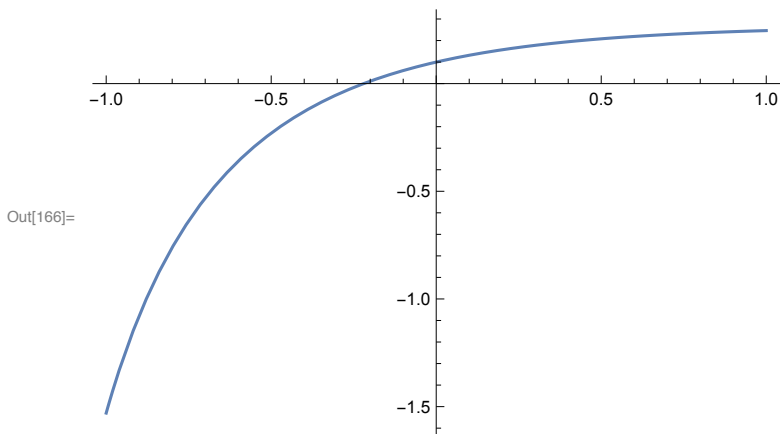
T[x_, a_] := b[a] / c[x, a]

In[164]:= **R2[x_, a_, w_] := R[x, a] + $\frac{T[x, a]^2 w}{1 - w R[x, a]}$**

In[165]:= **R2[x, a, w]**

Out[165]=
$$\frac{\text{Sinh}[x]}{\sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x]} + \frac{(-1 + a^2) w}{(\sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x])^2 \left(1 - \frac{w \text{Sinh}[x]}{\sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x]}\right)}$$

In[166]:= **Plot**[R[x, 2.0, 0.1], {x, -1, 1}]



In[167]:=

In[168]:= **FullSimplify**[R1[x, a, w] - R2[x, a, w]]

Out[168]= 0

In[169]:=

In[182]:= **Trace**[**Simplify**[R1[x, a, w] - R2[x, a, w]]]

Out[182]= $\left\{ \left\{ \left\{ R1[x, a, w], \frac{1 - w(a - b[a] \text{Coth}[x])}{a - w + b[a] \text{Coth}[x]}, \right. \right. \right.$
 $\left. \left\{ \left\{ \left\{ \left\{ b[a], \sqrt{a^2 - 1}, \{a^2 - 1, -1 + a^2\}, \sqrt{-1 + a^2}, \sqrt{-1 + a^2} \right\}, \sqrt{-1 + a^2} \text{Coth}[x] \right\}, \right. \right.$
 $\left. \left. - \left(\sqrt{-1 + a^2} \text{Coth}[x] \right), -\sqrt{-1 + a^2} \text{Coth}[x] \right\}, a - \sqrt{-1 + a^2} \text{Coth}[x] \right\},$
 $\left. \left. w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right) \right\}, - \left(w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right) \right) \right\},$
 $\left. \left. -w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right) \right\}, 1 - w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right) \right\},$
 $\left\{ \left\{ \left\{ \left\{ b[a], \sqrt{a^2 - 1}, \{a^2 - 1, -1 + a^2\}, \sqrt{-1 + a^2}, \sqrt{-1 + a^2} \right\}, \sqrt{-1 + a^2} \text{Coth}[x] \right\}, \right. \right.$
 $\left. \left. a - w + \sqrt{-1 + a^2} \text{Coth}[x] \right\}, \frac{1}{a - w + \sqrt{-1 + a^2} \text{Coth}[x]} \right\},$
 $\left. \left. \frac{1 - w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right)}{a - w + \sqrt{-1 + a^2} \text{Coth}[x]}, \frac{1 - w \left(a - \sqrt{-1 + a^2} \text{Coth}[x] \right)}{a - w + \sqrt{-1 + a^2} \text{Coth}[x]} \right\}, \right.$
 $\left. \left\{ R2[x, a, w], R[x, a] + \frac{T[x, a]^2 w}{1 - w R[x, a]}, \right. \right.$
 $\left. \left\{ R[x, a], \frac{\text{Sinh}[x]}{c[x, a]}, \left\{ c[x, a], a \text{Sinh}[x] + b[a] \text{Cosh}[x], \right. \right. \right.$
 $\left. \left\{ b[a], \sqrt{a^2 - 1}, \{a^2 - 1, -1 + a^2\}, \sqrt{-1 + a^2}, \sqrt{-1 + a^2} \right\}, \sqrt{-1 + a^2} \text{Cosh}[x] \right\},$
 $\left. \left. a \text{Sinh}[x] + \sqrt{-1 + a^2} \text{Cosh}[x], \sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x] \right\}, \right.$
 $\left. \left. \frac{1}{\sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x]} \right\}, \frac{\text{Sinh}[x]}{\sqrt{-1 + a^2} \text{Cosh}[x] + a \text{Sinh}[x]} \right\},$
 $\left\{ \left\{ T[x, a], \frac{b[a]}{c[x, a]}, \left\{ b[a], \sqrt{a^2 - 1}, \{a^2 - 1, -1 + a^2\}, \sqrt{-1 + a^2}, \sqrt{-1 + a^2} \right\}, \right. \right.$
 $\left. \left\{ c[x, a], a \text{Sinh}[x] + b[a] \text{Cosh}[x], \right. \right.$
 $\left. \left. \left\{ b[a], \sqrt{a^2 - 1}, \{a^2 - 1, -1 + a^2\}, \sqrt{-1 + a^2}, \sqrt{-1 + a^2} \right\}, \sqrt{-1 + a^2} \text{Cosh}[x] \right\}, \right.$

$$\begin{aligned}
& \frac{1 - w \left(a - \sqrt{-1 + a^2} \operatorname{Coth}[x] \right)}{a - w + \sqrt{-1 + a^2} \operatorname{Coth}[x]} + \\
& \left(- \frac{\operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} - \right. \\
& \left. \frac{(-1 + a^2) w}{\left(\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x] \right)^2 \left(1 - \frac{w \operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} \right)} \right), \\
& \frac{1 - w \left(a - \sqrt{-1 + a^2} \operatorname{Coth}[x] \right)}{a - w + \sqrt{-1 + a^2} \operatorname{Coth}[x]} - \frac{\operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} - \\
& \left. \frac{(-1 + a^2) w}{\left(\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x] \right)^2 \left(1 - \frac{w \operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} \right)} \right\}, \\
& \text{Simplify} \left[\frac{1 - w \left(a - \sqrt{-1 + a^2} \operatorname{Coth}[x] \right)}{a - w + \sqrt{-1 + a^2} \operatorname{Coth}[x]} - \right. \\
& \frac{\operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} - \\
& \left. \frac{(-1 + a^2) w}{\left(\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x] \right)^2 \left(1 - \frac{w \operatorname{Sinh}[x]}{\sqrt{-1 + a^2} \operatorname{Cosh}[x] + a \operatorname{Sinh}[x]} \right)} \right], 0 \}
\end{aligned}$$